Hypersonic Flow over Slender Double Wedges

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IN a recent paper, Bird[1] used numerical solutions to the equations of the interaction of two shock waves of the same family in order to determine the strength of the reflected waves generated by the inviscid supersonic and hypersonic flow over a double wedge. The author has obtained a simple approximation solution to the equations based on the hypersonic small disturbance theory, which is valid for finite strengths of the second shock. This approximate solution can be used to estimate the coefficients describing the strength of the reflected waves in the hypersonic regime, and an interesting comparison between the two methods can be made.

The inviscid hypersonic flow generated by a slender double wedge body is depicted in Fig. 1. The two shock waves AC and BC produced by successive wedges (angles ϕ_1 and ϕ_2, respectively) intersect to generate a third shock CD, a slip line CE, and a reflected wave CFG. At hypersonic speeds CFG is usually an expansion. Pressure variations will occur along the second wedge. Far upstream the pressure tends asymptotically to that generated by a single wedge of angle ϕ_D (Fig. 1).

If $\phi_D \ll 1$ and the freestream Mach number $(M_\infty) \gg 1$, then the following hypersonic small disturbance relations[2] apply across the shock AC:

$$M_\infty\sigma_1 = [M_\infty\phi_1/2(1-\epsilon)] + [\{M_\infty\phi_1/2(1-\epsilon)\}^2 + 1]^{1/2}$$
$$\epsilon = [(\gamma-1)/(\gamma+1)] \quad (1)$$

$$p_1/p_\infty = (1+\epsilon)(M_\infty\sigma_1)^2 - \epsilon \quad (2)$$

$$\left(\frac{M_1}{M_\infty}\right)^2 = \frac{(M_\infty\,\sigma_1)^2}{\{(1+\epsilon)(M_\infty\sigma_1)^2 - \epsilon\}\{\epsilon(M_\infty\sigma_1)^2 + (1-\epsilon)\}} \quad (3)$$

where p is the pressure, σ the shock angle, and γ the specific heat ratio. The subscripts denote the regions so marked in Fig. 1. A similar set of relations apply across the other shocks in the field. By using these relations, p_2, M_2, the asymptotic shock angle σ_D, and pressure p_D can be estimated.

Let ϑ be the angle of the initial part of the slip line CE relative to the second wedge. Then the pressure change across the expansion is given by[2]

$$p_4/p_2 = \{1 - [\epsilon/(1-\epsilon)]M_2\vartheta\}^{[(1+\epsilon)/\epsilon]} \quad (4)$$

The pressure generated by the shock CD is

$$p_3/p_\infty = (1+\epsilon)(M_\infty\sigma_3)^2 - \epsilon \quad (5)$$

where

$$M_\infty\sigma_3 = \left\{\frac{M_\infty(\phi_D+\vartheta)}{2(1-\epsilon)}\right\} + \left[\left(\frac{(\phi_D+\vartheta)M_\infty}{2(1-\epsilon)}\right)^2 + 1\right]^{1/2} \quad (6)$$

It is observed that $\vartheta/\phi_D \ll 1$. Hence, Eqs. (4–6) can be expanded in Taylor's series about $\vartheta = 0$ to yield

$$\frac{p_4}{p_\infty} = \frac{p_2}{p_\infty}\left[1 - \left(\frac{1+\epsilon}{1-\epsilon}\right)M_2\vartheta + \ldots\right] \quad (7)$$

$$\frac{p_3}{p_\infty} = \frac{p_D}{p_\infty} + 2\vartheta\left(\frac{p_D}{p_\infty}+\epsilon\right)\left\{\phi_D^2 + \left[\frac{2(1-\epsilon)}{M_\infty}\right]^2\right\}^{-1/2} \quad (8)$$

By using the matching condition $p_3 = p_4$ across the slip line, an equation for ϑ is obtained:

$$\vartheta = \frac{p_2 - p_D}{p_\infty}\left[\left(\frac{1+\epsilon}{1-\epsilon}\right)\frac{p_2}{p_\infty}M_2 + 2\left(\frac{p_D}{p_\infty}+\epsilon\right)\left\{\phi_D^2 + \left(\frac{2(1-\epsilon)}{M_\infty}\right)^2\right\}^{1/2}\right]^{-1} \quad (9)$$

By using this solution and Eq. (4), the reflection coefficients used by Bird

$$R_1 = \frac{p_2 - p_4}{p_2 - p_1} \qquad R_2 = \frac{p_2 - p_D}{p_2 - p_1} \quad (10)$$

can be calculated. In Fig. 2 the approximate values of R_1 and R_2 are given for $\phi_1 = \phi_2 = 10°$ and $\gamma = 1.4$ in the range $4 \leq M \leq 20$ and are compared with some exact values obtained by numerical solution of the full inviscid equations. The good agreement between the results obtained by the two methods demonstrates the usefulness of the hypersonic small disturbance theory when investigating this type of problem.

A point that has not been noted is that although $p_2 > p_D$, it is possible for the pressure on the second wedge to fall below p_D. The pressure p_5 just downstream of the reflection at the wall of the expansion wave can be obtained by using the simple wave equations. Usually $p_5 < p_D$. Therefore a measure of the effect of the shock interaction on the pressure distribution on the second wedge is more appropriately given by

$$R_3 = (p_2 - p_5)/(p_2 - p_1) \quad (11)$$

rather than R_2.

R_3 has been estimated using the approximate method for $\gamma = 1.4$ and $\phi_1 = \phi_2 = 10°$ and is given in Fig. 2. R_3 is larger than R_2 and the difference between them increases with increase in M_∞.

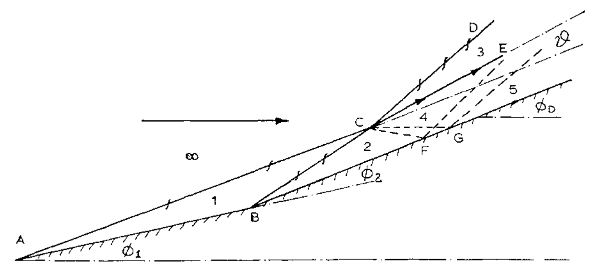

Fig. 1 Inviscid hypersonic flow over a slender double wedge.

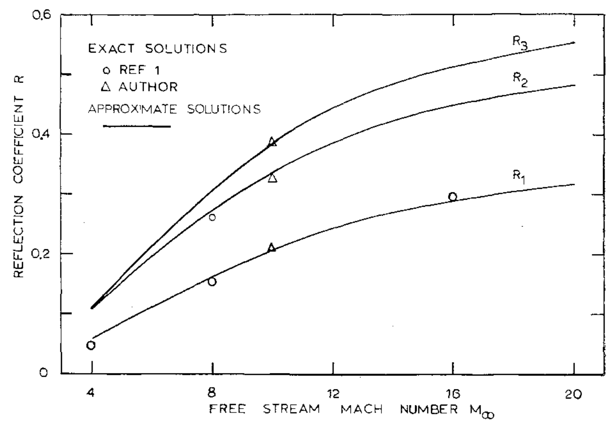

Fig. 2 Reflection coefficients for $\phi_1 = \phi_2 = 10°$ and $\gamma = 1.4$.

References

[1] Bird, G. A., "Effect of wave interactions on pressure distributions in supersonic and hypersonic flow," AIAA J. 1, 634–639 (1963).

[2] Chernyi, G. G., *Introduction to Hypersonic Flow* (Academic Press, New York, 1961), 41, 44.

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